

New Physics at Low Accelerations (MOND): an Alternative to Dark Matter

Mordehai Milgrom

The Weizmann Institute Center for Astrophysics, Rehovot 76100, Israel

Abstract. I describe the MOND paradigm, which posits a departure from standard physics below a certain acceleration scale. This acceleration as deduced from the dynamics in galaxies is found mysteriously to agree with the cosmic acceleration scales defined by the present day expansion rate and by the density of ‘dark energy’. I put special emphasis on phenomenology and on critical comparison with the competing paradigm based on classical dynamics plus cold dark matter. I also describe briefly nonrelativistic and relativistic MOND theories.

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“There can be no doubt that the interplanetary and interstellar spaces are not empty but are occupied by a material substance, or body, which is certainly the largest, and probably most uniform, body of which we have any knowledge.” (J. C. Maxwell).

“En astronomie, nous voyons les corps dont nous étudions les mouvements, et nous admettons le plus souvent qu’ils ne subissent pas l’action d’autres corps invisibles.” (H. Poincaré).

INTRODUCTION

Normal matter in the universe makes itself felt in many ways: It interacts through strong and electromagnetic interactions; so it emits and absorbs radiation. It self collides efficiently under astrophysical conditions, and dissipates energy. It produces magnetic fields that can be felt by particle acceleration, or by inducing synchrotron emission, for example. It decays and produces fast particles. It causes mechanical effects as in supernova explosions, or in ram-pressure stripping of gas in galaxies. In contradistinction, the putative, omnipresent dark matter (DM) is not required (and is not known) to do any of these things. The only trait it is required to share with standard matter is gravity: it is invoked only so as to modify and enhance the gravitational field of the visible baryons. It is thus conceivable that DM does not, in fact, exist, and that the extra gravity it is purported to supply is provided by a modification of the standard laws of dynamics.

So, following Poincaré’s admonition, and not Maxwell’s misguided conviction, we have been pursuing for a quarter of a century now a new paradigm of dynamics, dubbed ‘MOND’. Its main *raison d’être* remains the explanation of dynamics in the universe without the need for DM, which by and large, it does very well (with some exceptions). In particular, right at its advent it made a number of strong predictions, which have been confirmed over the years, and unearthed a number of unsuspected regularities in the properties of galactic systems. But, an additional motivation for embracing MOND has emerged with the recognition that the constant characterizing the paradigm may have cosmological origin, and that MOND as expressed in local physics has symmetries that may have descended from cosmology.

THE MOND PARADIGM

MOND starts by introducing into physics a new constant, a_0 , with the dimensions of acceleration. This constant marks the borderline between the pre-MOND physics—valid approximately for accelerations much larger than a_0 —and the MOND regime of much lower accelerations, just as \hbar does in the context of quantum physics, and c in that of relativity. The MOND constant also enters strongly into physics in the MOND regime and is predicted to appear in many apparently unconnected laws and relations in this regime of phenomena. To recover standard physics in the high

acceleration regime, we require that when we formally take $a_0 \rightarrow 0$ in all the equations of motion, a MOND theory tends to standard physics (like the restoration of classical predictions when $\hbar \rightarrow 0$ in quantum predictions for various observables). Finally, to achieve the phenomenological goals of MOND, we require that in the deep-MOND limit of *nonrelativistic* (NR) physics; i.e., formally taking $a_0 \rightarrow \infty$, and the masses $m \rightarrow 0$ so that ma_0 is fixed, the theory becomes scale invariant [55]; i.e., invariant under $(t, \mathbf{r}) \rightarrow \lambda(t, \mathbf{r})$. To see what this means, take, for example, a system of masses m_i with all accelerations much smaller than a_0 . Then, if $\mathbf{r}_i(t)$ describes some possible history of the system, $\lambda \mathbf{r}_i(t/\lambda)$ is also a solution of the theory. For example, the length of the planetary year in a deep-MOND planetary system is proportional to the orbital size, not to its $3/2$ power as in the Newtonian Kepler's 3rd law.

Look, indeed, at the quintessential problem of the circular motion of a test particle at a distance R around a point mass M . On dimensional grounds alone, the expression for MOND acceleration of the particle must be of the form

$$a = \frac{MG}{R^2} v \left(\frac{MG}{R^2 a_0} \right). \quad (1)$$

The MOND basic tenets require that for $MGR^{-2} \gg a_0$ we have $v \rightarrow 1$, while for $MGR^{-2} \ll a_0$ we have to have scale invariance, which dictates $v(y) \approx y^{-1/2}$ (the normalization is absorbed in the definition of a_0), or

$$a \approx (MGA_0)^{1/2}/R. \quad (2)$$

There are three main points to take home from this simple, introductory case, which are also relevant to more general systems: 1. For a given mass, M , the asymptotic acceleration at large radii goes as R^{-1} compared with the standard R^{-2} (accelerations scale as λ^{-1}). 2. In this region, a is proportional to $M^{1/2}$, not to M . 3. The transition occurs always at the same value of the acceleration (not a fixed radius).

Extensive reviews of various aspects of the MOND paradigm can be found in [73, 76, 7, 54, 80].

MOND THEORIES

The above basic tenets of MOND can be incorporated into various MOND theories. For example, in the NR regime one can modify Newtonian gravity by generalizing the Poisson equation for the gravitational field [9, 58]. Another option, which cannot be described as modified gravity is to modify the kinetic action of particles leading to modification of Newton's second law, or to modified inertia [47].

Nonrelativistic theories

The action governing a Newtonian system made of a gravitating mass distribution $\rho(\mathbf{r})$, which produces a gravitational potential ϕ , can be written as $I = \int (\mathcal{L}_K + \mathcal{L}_P) d^3\mathbf{r} dt$, with $\mathcal{L}_K = \rho \mathbf{v}^2/2$ (\mathbf{v} is the velocity field) and $\mathcal{L}_P = -(8\pi G)^{-1}(\vec{\nabla}\phi)^2 - (1/2)\rho\phi$. Bekenstein and Milgrom [9] generalized this Lagrangian by replacing $(\vec{\nabla}\phi)^2$ with $a_0^2 \mathcal{F}[(\vec{\nabla}\phi)^2/a_0^2]$ to obtain a MOND generalization of the Poisson equation

$$\vec{\nabla} \cdot [\mu(|\vec{\nabla}\phi|/a_0) \vec{\nabla}\phi] = 4\pi G\rho, \quad (3)$$

with $\mu(x) = \mathcal{F}'(x^2)$ satisfying $\mu(x \gg 1) \approx 1$, $\mu(x \ll 1) \approx x$. This theory had been, for many years, the only complete MOND theory in stock. It has been applied extensively to many problems, both analytically and numerically. For example, solar system tests [44, 8, 57], forces on massive bodies [48, 51, 26], disc stability and bar formation [19, 85], two-body relaxation [23], dynamical friction [64], escape speed from a galaxy [29, 88], galaxy interactions (Combes and Turet, this volume, [84, 62]), galaxy collapse [63], triaxial models of galactic systems [87, 89], the external-field effect as applied to dwarf spheroidals and warp induction [20, 21, 1], structure formation (e.g., [38]), and quite a few more.

This formulation of MOND has also been interpreted as resulting for the omnipresence of a gravitationally polarizable medium [10], with a relativistic generalization [11, 12].

It has been realized recently [58] that a larger family of modified-gravity MOND theories exist, which involve two or even more potentials, with only one of them coupling to matter. For example, start from a Palatini-type formulation of

Newtonian gravity, introducing beside ϕ an acceleration field $\tilde{\mathbf{g}}$, and taking instead of the above Poissonian Lagrangian density:

$$\mathcal{L}_p = \frac{1}{8\pi G}(\tilde{\mathbf{g}}^2 + 2\vec{\nabla}\phi \cdot \tilde{\mathbf{g}}) - \frac{1}{2}\rho\phi. \quad (4)$$

It gives, upon variation over $\tilde{\mathbf{g}}$ and ϕ : $\tilde{\mathbf{g}} = -\vec{\nabla}\phi$, and $\vec{\nabla} \cdot \tilde{\mathbf{g}} = -4\pi G\rho$, respectively, which is indeed standard Newtonian gravity. If we generalize this action by replacing $\tilde{\mathbf{g}}^2$ with $a_0^2 \mathcal{Q}[\tilde{\mathbf{g}}^2/a_0^2]$ we get a theory that is equivalent to that described by eq.(3). However, if we require a priori that $\tilde{\mathbf{g}}$ is derivable from an auxiliary potential $\tilde{\mathbf{g}} = -\vec{\nabla}\tilde{\phi}$, we get a new theory that is a quasi-linear MOND (QUMOND) theory, which is rather easier to apply because it requires solving only linear differential equations [58]

$$\Delta\tilde{\phi} = 4\pi G\rho, \quad \Delta\phi = \vec{\nabla} \cdot [\nu(|\vec{\nabla}\tilde{\phi}|/a_0)\vec{\nabla}\tilde{\phi}], \quad (5)$$

with $\nu(y) \equiv \mathcal{Q}'(y^2)$. This theory requires solving the Poisson equation twice, with a nonlinear algebraic step in between. It has been used recently, beside eq.(3), to calculate MOND effects in the inner solar system [57].

We can get even a larger class of theories by making \mathcal{L}_p a function of the three scalars $(\vec{\nabla}\phi)^2$, $(\vec{\nabla}\tilde{\phi})^2$, $\vec{\nabla}\phi \cdot \vec{\nabla}\tilde{\phi}$ (and possibly involving even more potentials) [58]. Look, for example, at actions of the form

$$\mathcal{L} = -\frac{1}{8\pi G}\{\beta(\vec{\nabla}\phi)^2 + \alpha(\vec{\nabla}\tilde{\phi})^2 - a_0^2\mathcal{M}[(\vec{\nabla}\phi - \vec{\nabla}\tilde{\phi})^2/a_0^2]\} + \rho(\frac{1}{2}\mathbf{v}^2 - \phi), \quad (6)$$

which leads to the field equations

$$\vec{\nabla} \cdot [\mu^*(|\vec{\nabla}\tilde{\phi}|/a_0)\vec{\nabla}\tilde{\phi}] = 4\pi G\rho, \quad \Delta\phi = \vec{\nabla} \cdot [(1 - \alpha^{-1}\mathcal{M}')\vec{\nabla}\tilde{\phi}], \quad (7)$$

with

$$\tilde{\phi} \equiv \phi - \hat{\phi}, \quad \mu^* = \beta - \frac{\alpha + \beta}{\alpha}\mathcal{M}'[(\vec{\nabla}\tilde{\phi}/a_0)^2]. \quad (8)$$

The first equation in (7) is solved for $\tilde{\phi}$, and then the second is a Poisson equation for the MOND potential ϕ , with the known right hand side as source. The parameter range $0 < \beta + \alpha \leq 1$ is excluded. The limiting case $\beta = -\alpha = 1$ is particularly simple, as the theory then reduces to the QUMOND theory (5) with $\nu(y) = 1 + \mathcal{M}'(y^2)$. These theories have lead to a class of relativistic, bimetric MOND (BIMOND) theories (see below).

Modified inertia

An altogether different route to constructing MOND theories(e.g., [47, 54]) is to replace the kinetic action $\sum_i m_i \mathbf{v}_i^2/2$ (written now for a collection of point masses) by a more general functional of the trajectories. Such theories have, generically, to be time nonlocal if they are to obey Galilei invariance and the proper Newtonian and MOND limits [47].

Interestingly, such theories predict a universal equation that determines the orbital motion on circular trajectories in axisymmetric potentials, relating the orbital speed, V , and radius, R , by

$$\mu(V^2/Ra_0)V^2/R = -\frac{\partial\phi_N}{\partial R}, \quad (9)$$

where ϕ_N is the *Newtonian* potential, and $\mu(x)$ is a function that derives from the action of the theory as specialized to circular, constant-speed orbits. This relation is unique for a given theory (i.e., system independent). All MOND rotation-curve analyses to date employ this relation [not the relation derived from theories such as eq.(3) or eq.(5), which would require a new numerical calculation for each system analyzed].

Such ‘modified-inertia’ theories can differ greatly from modified-gravity formulations of MOND concerning some phenomena, even in the NR regime. Whereas in the latter, the anomalous MOND acceleration of test particles in the field of a given mass depends only on the position in the (modified) field, the time nonlocality of the former theory can produce retardation and hysteresis effects, and makes the anomalous acceleration at any location depend on properties of the whole orbit. For example, if the accelerations are small on some segments of a trajectory, MOND effects can be felt also on segments where the accelerations are high. This can give rise, for example, to different MOND effects on bound and unbound orbits, or on circular and highly elliptic orbits. In the solar system this can differentiate between

planets on one hand, and long period comets and unbound spacecraft, such as the Pioneers, on the other (Milgrom, in preparation).

Also, in such theories the predicted behavior of different MOND effects, such as the external-field effect (EFE), can be rather different from that in modified-gravity theories (Milgrom, in preparation).

Relativistic theories

One naturally wants to incorporate the MOND principles in a relativistic extension. The most studied theory following this effort is Bekenstein's Tensor-Vector-Scalar (TeVeS) theory [6], which is an outgrowth of Sanders's stratified gravity approach [69]. There is a large body of work elaborating on, extending, reinterpreting, and criticizing TeVeS. See, e.g., [7, 90, 91, 22, 24, 67, 66], and, in particular, see the extensive reviews [7, 80].

There is also initial investigation of relativistic bimetric MOND (BIMOND) theories, brought to light recently [59], which I describe succinctly below.

All such theories, even if they turn out to work well phenomenologically, must be only effective, approximate theories as evinced by the appearance of a free function in them, which is to reproduce the interpolating function in equations such as eq.(1). The exact form of this function will hopefully some day emerge from a deeper theory underlying the effective ones.

TeVeS

Here are the main features of this much discussed theory.

- Gravity in TeVeS [6] is described by a metric $g_{\alpha\beta}$, as in General Relativity (GR), plus a vector field, \mathcal{U}_α , and a scalar field ϕ . (In other formulations the scalar is eliminated [90].)
- Matter is coupled to one combination of the fields: the physical metric $\tilde{g}_{\alpha\beta} \equiv e^{-2\phi}(g_{\alpha\beta} + \mathcal{U}_\alpha \mathcal{U}_\beta) - e^{2\phi} \mathcal{U}_\alpha \mathcal{U}_\beta$.
- $g_{\alpha\beta}$ is governed by the usual Hilbert-Einstein action, the vector field (constrained to have a unit length)¹ by a Maxwell-like action,¹ and the scalar action can be written as $S_s = -\frac{1}{2Gk\hat{k}} \int Q[\hat{k}(g^{\alpha\beta} - \mathcal{U}^\alpha \mathcal{U}^\beta) \phi_{,\alpha} \phi_{,\beta}] (-g)^{1/2} d^4x$.
- There are three constants: k , \hat{k} , and a parameter K appearing in the vector action, and one free function $Q(x)$, which engenders the interpolating function of MOND in the NR limit.
- For NR systems TeVeS reproduces the NR MOND phenomenology with the MOND potential being the sum of two potentials, one satisfying the (linear) Poisson equation, the other satisfying eq.(3) with $a_0 \propto k\hat{k}^{-1/2}$.
- For weak fields ($\phi \ll c^2$) TeVeS gives lensing according to the standard GR formula but with the MOND potential.
- Structure formation and CMB: According to preliminary work, TeVeS has the potential to mimic aspects of cosmological DM [71, 27, 78, 81, 92].

Bimetric MOND gravity

Inspired by bi-potential theories governed by the Lagrangian density (6), the BIMOND theories [59] involve two metrics as independent degrees of freedom. One, $g_{\mu\nu}$, is the MOND metric, which alone appears in the matter action, and which couples in the standard way to matter. The other metric, $\hat{g}_{\mu\nu}$, is an auxiliary one. In constructing an action for the theory we now have at our disposal the usual scalars made of the curvature tensors of the two metrics such as the Ricci scalars R and \hat{R} . But, in addition we can construct scalars using the tensor difference between the Levi-Civita connections of the two metrics:

$$C_{\beta\gamma}^\alpha = \Gamma_{\beta\gamma}^\alpha - \hat{\Gamma}_{\beta\gamma}^\alpha. \quad (10)$$

This is particularly germane in the context of MOND: Connections, and hence the $C_{\beta\gamma}^\alpha$, play the role of gravitational accelerations. So, without introducing new constants in the relativistic context we can use the MOND constant a_0 to

¹ This was later modified to obviate some inconsistencies in the original version [79].

form dimensionless tensors, $a_0^{-1}C_{\beta\gamma}^\alpha$, and from these construct scalars.² Functions of these scalar can then be used as Lagrangian densities.

Examples of scalars that are quadratic in $C_{\beta\gamma}^\alpha$ are

$$\Upsilon_i^{(2)} = g^{\mu\nu}C_{\mu\lambda}^\gamma C_{\nu\gamma}^\lambda, \quad \bar{C}^\gamma C_\gamma, \quad g_{\mu\nu}\bar{C}^\mu \bar{C}^\nu, \quad g^{\mu\nu}C_\mu C_\nu, \quad g_{\alpha\lambda}g^{\beta\mu}g^{\gamma\nu}C_{\beta\gamma}^\alpha C_{\mu\nu}^\lambda, \quad (11)$$

where $\bar{C}^\gamma \equiv g^{\mu\nu}C_{\mu\nu}^\gamma$, $C_\gamma \equiv C_{\gamma\nu}^\nu$, as well as others, e.g., contractions with $\hat{g}_{\mu\nu}$, in addition to higher, m powers $\Upsilon_i^{(m)}$. We thus seek to construct a covariant Lagrangian density that is a function of R , \hat{R} , and scalars Υ_i . I take the Lagrangian to be linear in the curvature scalars, so as not to end up with a higher derivative theory. The tensor $C_{\beta\gamma}^\alpha$ and the scalars Υ_i contain only first derivatives of the metrics, so we can take functions of them that interpolate between the MOND ($\Upsilon_i^{(2)}/a_0^2 \ll 1$) and standard ($\Upsilon_i^{(2)}/a_0^2 \gg 1$) regimes.

I have thus considered [59] actions of the form

$$I = -\frac{1}{16\pi G} \int [\beta g^{1/2}R + \alpha \hat{g}^{1/2}\hat{R} - (g\hat{g})^{1/4}f(\kappa)a_0^2\mathcal{M}(\Upsilon/a_0^2)]d^4x + I_M(g_{\mu\nu}, \psi_i) + \hat{I}_M(\hat{g}_{\mu\nu}, \chi_i), \quad (12)$$

where for brevity's sake I write \mathcal{M} as a function of only one quadratic scalar. Its derivative \mathcal{M}' plays the role of an interpolating function between the MOND and conventional regime. Also, I_M is the matter action, with matter degrees of freedom represented by ψ_i , coupling only to $g_{\mu\nu}$ [g and \hat{g} are minus the determinants of the two metrics, and $\kappa = (g/\hat{g})^{1/4}$]. I also permit twin matter (TM) described by degrees of freedom χ_i , which couples only to $\hat{g}_{\mu\nu}$.

We see that the modification of GR entailed by MOND does not enter here by modifying the ‘elasticity’ of space-time (except perhaps its strength), as is done in $f(R)$ theories and the like. The modification is introduced through the interaction between the space-time on which ‘our’ ψ matter lives and the auxiliary one. In a membrane description of gravity, we can say that matter lives on one membrane of a pair, with the two membranes, each with its own standard elasticity, coupled together. The way the shape of the home membrane is affected by matter then depends on the combined elasticity properties of the double membrane. However, matter response depends only on the shape of its home membrane. Such heuristics may, in fact, lead to a fundamental understanding of the origins of the MOND paradigm, and the meaning of the length $\ell = c^2/a_0$, which enters NR physics as a_0 .

In particular, I found scalar arguments of \mathcal{M} constructed from the tensor $\Upsilon_{\mu\nu} = C_{\mu\lambda}^\gamma C_{\nu\gamma}^\lambda - C_{\mu\nu}^\gamma C_{\lambda\gamma}^\lambda$ to lead to simple theories. For example $g^{\mu\nu}\Upsilon_{\mu\nu}$, or $\hat{g}^{\mu\nu}\Upsilon_{\mu\nu}$ (or a symmetric combination of both). Such a tensor is constructed from $C_{\beta\gamma}^\alpha$ in the same way as the first-derivative part in the Ricci tensor is constructed from the connections. Also, the subclass of theories with $\beta + \alpha = 0$, and arbitrary scalar argument, is interesting and simple, and has been studied in more detail.

Such theories have the following properties:

- Beside Newton’s G (and c) they involves only a_0 as a new constant.
- The equations of motion involve no higher than second derivatives in the metrics.
- The nonrelativistic limit: On a locally double-Minkowski background, the first order NR metric for any α , β is given by $g_{\mu\nu} = \eta_{\mu\nu} - 2\phi\delta_{\mu\nu}$, where ϕ is the MOND potential that is determined by the theory given by eq.(7), and $\hat{g}_{\mu\nu} = \eta_{\mu\nu} - 2\hat{\phi}\delta_{\mu\nu}$, with $\phi^* = \phi - \hat{\phi}$ being the Newtonian potential. QUMOND is gotten for $\beta = -\alpha = 1$. I also considered backgrounds other than double Minkowski.
- Gravitational lensing by slowly moving masses: This has always been a holy grail for relativistic MOND theories. It is thus reassuring that the BIMOND theories predict enhanced, MOND-like lensing. Since the relation between the full first order MOND metric and the MOND potential is the same as the relation between the GR metric and the Newtonian potential, the MOND potential controls lensing exactly as the Newtonian potential does in GR. In other words, lensing and massive-particle analysis (assuming GR) of the gravitational field of a NR mass (e.g., a galaxy) would give the same MOND potential. With choices of the scalar argument Υ different from the above this result can change somewhat; but, lensing is still enhanced and MOND-like, i.e., underpinned by an asymptotic logarithmic potential $\propto (MGa_0)^{1/2}\ln(r)$.
- The GR limit: We can choose the form of \mathcal{M} so that in the formal limit $a_0 \rightarrow 0$ we obtain GR exactly, possibly with a ‘dark energy’ term of order $\mathcal{M}(\infty)a_0^2$. For instance, in the case $\beta = 1$ (and G the Newton constant) the requirement is that $\mathcal{M}'(z) \rightarrow 0$ for $z \rightarrow \infty$. This limit can be approached as fast as required; so departures from GR in the inner solar system and short period binary pulsars can be made as small as desired.

² I work in units where $c = 1$, otherwise we use the MOND scale-length $\ell = c^2/a_0$ in the dimensionless tensors $\ell C_{\beta\gamma}^\alpha$.

- Pinpointing the correct cosmology is still moot as it requires additional assumptions on initial conditions, symmetries, matter content (especially that of the twin matter), etc.. There exist, however, interesting cosmological solutions. For example, with a certain symmetry between the two types of matter there are cosmologies with $\hat{g}_{\mu\nu} = g_{\mu\nu}$ with $g_{\mu\nu}$ describing a standard GR cosmology (for G/β the Newton constant) with a cosmological constant $\Lambda = -\mathcal{M}(0)a_0^2/2(\alpha + \beta)$. Such cosmologies may be described with the two ‘membranes’ being stuck together ($\hat{g}_{\mu\nu} = g_{\mu\nu}$) on large scales, while they separate locally ($\hat{g}_{\mu\nu} \neq g_{\mu\nu}$) due to matter (and twin matter) inhomogeneities.

More generally, there appears in these theories a weakly variable ‘dark energy’ term that is of order $a_0^2\mathcal{M}$, which is of order a_0^2 if \mathcal{M} is of order unity. For example, a de Sitter, or Anti de Sitter, universe is a generic vacuum solution of these theories with cosmological constant $\sim \mathcal{M}(0)a_0^2$.

By and large, cosmological considerations seem to prefer a matter-twin-matter symmetric universe, with $\alpha = \beta = 1$. In this case $C_{\beta\gamma}^\alpha = 0$ in cosmology, and the cosmological equations reduce to those of GR with a cosmological constant.

Thus even without any connection with MOND, such theories may provide alternatives to ‘dark energy’, to be investigated alongside existing schemes, such as $f(R)$ theories.

- The theory has the (yet unproven) potential to account for all aspects of the ‘dark sector’ (galactic ‘DM’, cosmological ‘DM’, and ‘dark energy’) in one fell swoop, with all controlled by a_0 .
- All vacuum solutions $g_{\mu\nu}$ of GR [with $CC \sim a_0^2\mathcal{M}(0)$] are also vacuum solutions of BIMOND with $\hat{g}_{\mu\nu} = g_{\mu\nu}$. In particular, GR gravitational waves, in vacuum, are also BIMOND solutions. Also, a double Schwarzschild geometry is a vacuum solution of BIMOND (corresponding to an equal matter-TM central masses).

Another avenue for exploration brought to mind by these theories starts from the interesting possibility that twin matter exists, and then interacts ‘gravitationally’ with normal matter, indirectly through the \mathcal{M} term.³ Preliminary study [60] shows this effective interaction to be different from standard gravity, and rather peculiar, in ways that depend on the BIMOND version at hand. For example, in versions that are completely symmetric in the two types of matter, gravity within each sector is described by standard MOND, but matter and twin matter repel each other. The repulsion is MOND like in the MOND regime but the interaction disappears altogether at high accelerations. This would have obvious ramifications for structure formation, for example, as it would lead, among other things, to matter-twin-matter segregation.

Be all this promising as it may, it represents only initial work, and there are still important aspects of the theory to be checked. These include questions of causality, existence of ghosts, etc.. Bimetric theories have a long history and there is a considerable body of related literature discussing, among other things, the above questions of principle for various classes of bimetric theories. (See, for example, the recent treatment in [13, 14, 2], where the authors consider bimetric coupling involving the metrics themselves, not their derivatives, as in BIMOND theories.) In particular, it was shown [17] that bimetric theories with derivative coupling, satisfying some general assumptions, suffer generically from the existence of ghosts. It remains to be seen, however, how relevant and/or deleterious such results are to BIMOND, especially since a major assumption underlying them does not apply in BIMOND.

MOND PHENOMENOLOGY

A MOND theory should predict the motions of test and non-test bodies in the field of an arbitrary mass distribution, such as a galaxy. For example, rotation curve analysis of disc galaxies partly tests such predictions, as it probes the accelerations only in the symmetry plane of the disc and only for nearly circular orbits. However, beyond such tests of MOND predictions for specific, individual systems, MOND predicts a considerable number of relations that should hold between galaxy properties, and which are the analogous, in the realm of the galaxies, to Kepler’s laws of planetary motions. In such MOND laws, a_0 appears in several roles, similar to the appearance of \hbar in many quantum phenomena, such as the black-body spectrum, the photoelectric effect, atomic spectra, superconductivity, etc., or the appearance of the speed of light in relativistic phenomena, such as black-hole physics, time dilation, or the relation between velocity and momentum. Without the unifying force of the underlying theory the appearance of the same constant in disparate, and apparently unrelated, phenomena does not make sense.

³ It then does not play the role of the putative dark matter; this role is still played by the MOND departure from GR.

MOND laws of galactic motion

I now list briefly some of these predicted MOND laws of galactic motions. These, along with several others, are discussed, explained, and referenced in more detail in other publications on MOND (e.g. [54]).

- The rotational speed around an isolated mass becomes constant at large radii (asymptotic flatness of rotation curves): $V(r) \rightarrow V_\infty$.
- V_∞ depends only on the total ('baryonic') mass of the body via $V_\infty^4 = M G a_0$. This predicted the so called 'baryonic Tully-Fisher relation' [41, 83, 86] (see Fig.1).
- For quasi-isothermal systems, such as elliptical galaxies, the velocity dispersion depends only on the total mass (in contradistinction with the Newtonian virial relation) via $\sigma^4 \sim M G a_0$. This underlies the Faber-Jackson relation for elliptical galaxies.
- The mass discrepancy (transition from baryon dominance to 'DM' dominance) appears always around $V^2/R = a_0$.
- Isothermal spheres (e.g., as models of ellipticals) have mean surface densities $\bar{\Sigma} \leq a_0/G$, as is well supported by the data (see, e.g., Figs. 1, 2 in [34]).
- The central surface density of phantom 'dark halos' is $\approx a_0/2\pi G$ [56, 33], in accordance with the findings in [28].
- Discs with $\bar{\Sigma} \leq a_0/G$ have added stability.
- Disc galaxies should exhibit a distinct disc component of 'DM', with predicted properties, in addition to the extended, spheroidal 'DM' component [50], as deduced by [35].
- Interpreting MOND with 'DM' will result in negative DM densities in some, well specified, locations.

Several important facts about these predictions have to be noted: (i) They follow as inevitable consequences of the basic MOND tenets, and are oblivious to the exact way(s) in which galaxies or other galactic systems formed. (ii) They are independent predictions in the sense that without MOND none of them follows from the others.⁴ (iii) Inasmuch as they have been tested they are well consistent with the data. (iv) In the framework of DM such laws must follow from very strict connection between the amount and distribution of DM and those of the baryons, because they relate baryonic properties—such as (baryonic) masses—to properties determined mainly by the DM—such as speeds. (v) Even without further theoretical development, or interpretation, MOND has already directed the eye to many regularities not suspected before, including the appearance of an acceleration constant, a_0 , in many a priori unrelated facets of galaxy dynamics.

I now discuss briefly some of these laws: The independence of speed on orbital radius in the limit of large radii follows directly from the scale invariance of the deep-MOND limit: under space-time scaling sizes change, but velocities do not.

The MOND mass-asymptotic-speed relation, underlies the Tully-Fisher relation (TFR), whose existence was known before the advent of MOND. However, the traditional TFR correlates some luminosity measure of galaxies with some velocity measure, with different choices giving different results. To my knowledge the TFR has no creditable theoretical basis in the DM doctrine. MOND has specified exactly what is to be correlated with what: the total baryonic mass with the asymptotic, constant rotational speed. This has lead to the so-called baryonic TF correlation, which indeed is very tight, and which conforms exactly with the predicted MOND relation: the power of 4 and the proportionality factor being $G a_0$, well consistent with other determinations of a_0 [41, 40, 83]. Figure 1 shows the results of one such analysis.

The MOND constant, a_0 , defines a scale of mass surface density a_0/G . This is why several of the MOND laws predict a special value of the surface density in different contexts. For example, one of the above laws, which follows from detailed analysis, states that isothermal spheres—which in MOND have a finite mass, unlike their Newtonian analogs—cannot have mean surface densities much exceeding a_0/G . This simply reflects the fact the IS disobeying this inequality have mean accelerations larger than a_0 , and so are Newtonian, and so cannot exist as finite mass objects.

This last MOND prediction assigns a special surface density to the normal matter component of a galactic object. But there is also an interesting MOND prediction that has come to light recently, which pertains to a property of the pure, fictitious 'DM halo' needed to explain MOND results with DM. This prediction identifies $a_0/2\pi G$ as a special central

⁴ It is possible to build families of galaxies of baryons plus DM that will satisfy any subset of these laws, but not the rest. So in the framework of DM they'll each require a separate explanation.

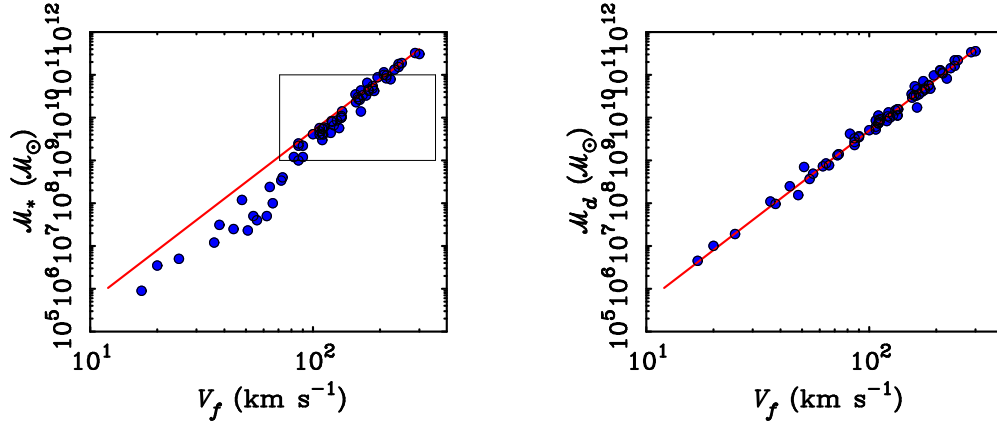


FIGURE 1. Galaxy mass plotted against the rotation curve plateau velocity. Left: analog of the traditional Tully-Fisher plot with mass in stars only. Right: The total mass including that of gas. The solid line has the log-log slope of 4, predicted by MOND, and is not a fit [40] (the small rectangle shows where past analysis had concentrated).

surface density of such phantom halos that is of universal significance [56, 33]. The discovery of this prediction was incited by a recent finding to this effect [28]. The two predictions follow from completely different aspects of MOND, but both come up with a special surface density determined by a_0/G , which is the only scale available in MOND. The fact that the same value of a_0 determined from other predictions gives a very good agreement with the findings of [28], can count as another success of MOND.

Rotation Curves of Disc Galaxies

The quintessential MOND achievement, however, is the prediction of detailed rotation curves (RCs) of many individual galaxies, from their baryon distribution alone. Interestingly, the first systematic analysis of such predictions appeared only some four years after the advent of MOND, for it had had to await the appearance of extended HI rotation curves. The first systematic study [36] was followed by amending analysis in [46]; then, many such analyses followed; e.g., [4, 68, 75, 39, 15, 16, 5, 30, 32, 25, 3, 74, 61].

I show in Fig.2 examples of such analysis for five disc galaxies that span the gamut of galaxy properties: from low-mass, low-speed, gas-rich galaxies with a still rising RC, to high-mass, high-speed, gas-poor galaxies, with a still declining RC.

The MOND curves shown involve only one parameter, the mass-to-light ratio, M/L , which is the conversion factor from stellar light to mass (the gas mass is measured directly with no need for conversion). In cases where the gas strongly dominates the mass budget, such as for the galaxy shown on the lower left, the MOND prediction hardly depends on the assumed M/L value, and the MOND curve is practically an absolute prediction. When the stars dominate, M/L may be viewed as a parameter chosen to fit the first few points on the rotation curve; the rest of the curve is then predicted uniquely. Note also, that M/L is not a completely free parameter: it can be deduced for a galaxy of a given spectrum from theoretical calculations of stellar population synthesis, within some margins. The values required by the MOND fits were found to agree with those theoretical values (e.g., [75]). In DM fits one usually has full freedom to adjust the radial scale and the normalization of the a priori unknown contribution of the DM halo, in addition to the stellar M/L value.

We see that some measured RCs exhibit features—such as a dip, or a sudden rise—which are well reproduced by the predicted MOND curves. These features are of prime importance in assessing the performance of competing paradigms. They result from relatively sharp features in the baryonic mass distribution of the disc. As this is the only source for the MOND curve, the feature appears in this curve in full strength (as it does in the Newtonian curve without DM). However, adding a Dominant DM component, which does not have the feature (indeed, which cannot have the feature if it is spherical) washes the feature out in the best-fit DM curve.

Figures 3, 4 show more results of MOND rotation-curve analyses.

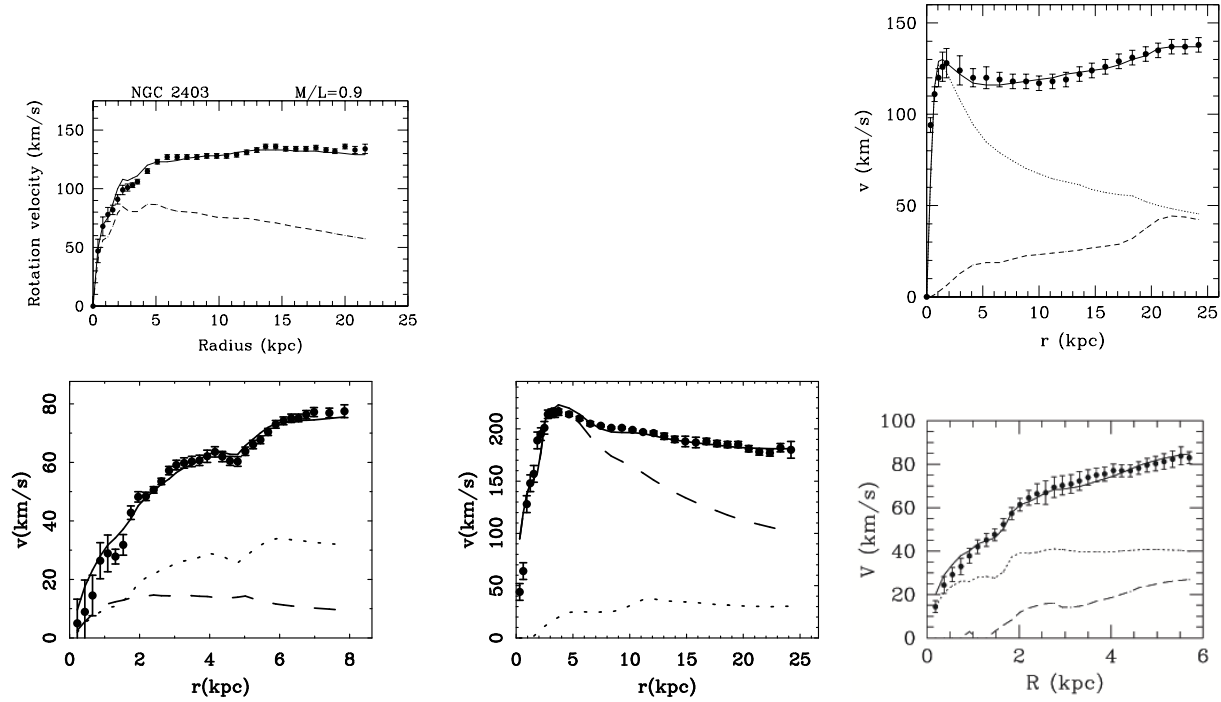


FIGURE 2. The rotation curves of a variety of galaxies from low-mass-low-speed galaxies that are everywhere of low-acceleration (such as the galaxies shown at the lower right and left) to high-mass-high-speed galaxies that have high accelerations in the inner parts transiting to low accelerations at a finite radius (such as the middle plot). The data is shown as points with error bars. The line through the data is the MOND curve. Upper left (courtesy of Bob Sanders): dashed line shows the predicted Newtonian curve fitted to match the inner points. Lower left (from [73]): dotted curve is the Newtonian curve calculated for the gas alone, and the dashed line the same for the stars (they add in quadrature). Middle (from [73]): dashed line the Newtonian curve for the stars, and the dotted curve for the gas. Upper right (from [72]) and Lower right (courtesy of Bob Sanders): the dotted line is the Newtonian curve for the stars, the dashed curve for the gas.

Round systems

Dwarf spheroidal satellites of the Milky Way are very low acceleration systems. They indeed show large mass discrepancies, as was predicted from MOND[43] before these discrepancies were measured. The state of the art analysis of these systems and references to earlier work can be found in [1, 77], where it is found that, with one or two exceptions, perhaps, MOND indeed accounts for the large mass discrepancies deduced with Newtonian dynamics.

As has been realized through many analyses starting in the early 1990s, the only system type where MOND has failed systematically to account for the full mass discrepancy comprise galaxy clusters. The situation has been reviewed recently in [53] (and many references therein). Briefly: the typical global mass discrepancy in clusters (within say 2 Mpc) is about a factor 7-10. MOND corrects this by reducing the required mass by a factor 3-5, but leaves still a factor of two discrepancy. So roughly as much ‘dark mass’ is still required as there is presently visible baryonic mass. MOND adherents have attributed this dark mass to possibly neutrinos or to some form of yet undetected cluster baryonic matter (CBDM). The distribution of the CBDM in the cluster is rather like that of the galaxies, being rather more centrally concentrated than the hot gas, which is makes the lion’s share of the visible component. This makes the mass discrepancy that remains after the MOND correction rather stronger in the cluster core (within a few hundred kpc). The appearance of the post-collision ‘bullet cluster’, which has been bruited as ‘direct proof of the existence of DM’, conforms exactly to what is expected from this picture, long known from studies of individual clusters. It only

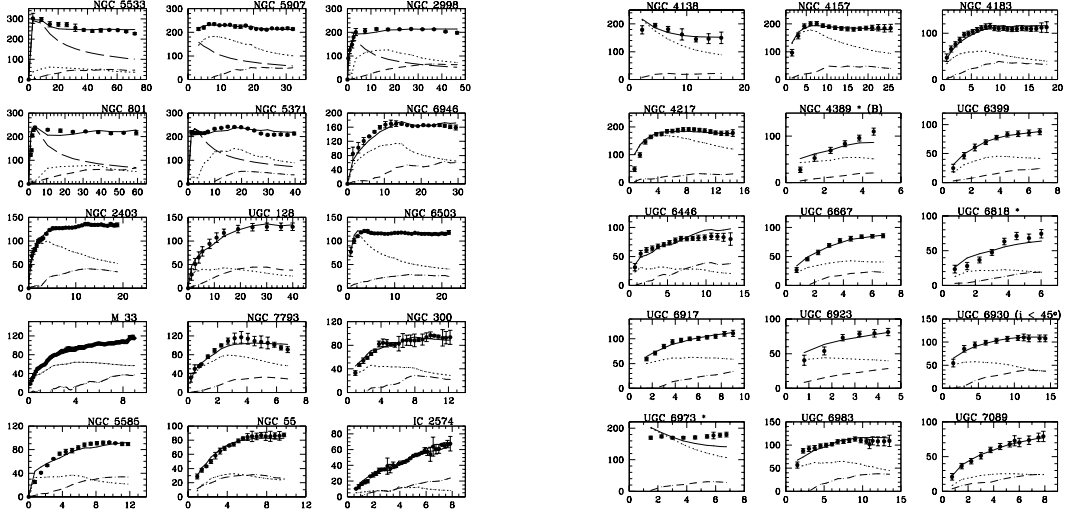


FIGURE 3. Additional MOND rotation curves from [68] and [15] (left) and from [75] (right). (MOND curves in solid; stellar disc Newtonian curves in dotted; gas in dot-dash; and stellar bulge in long dashed.)

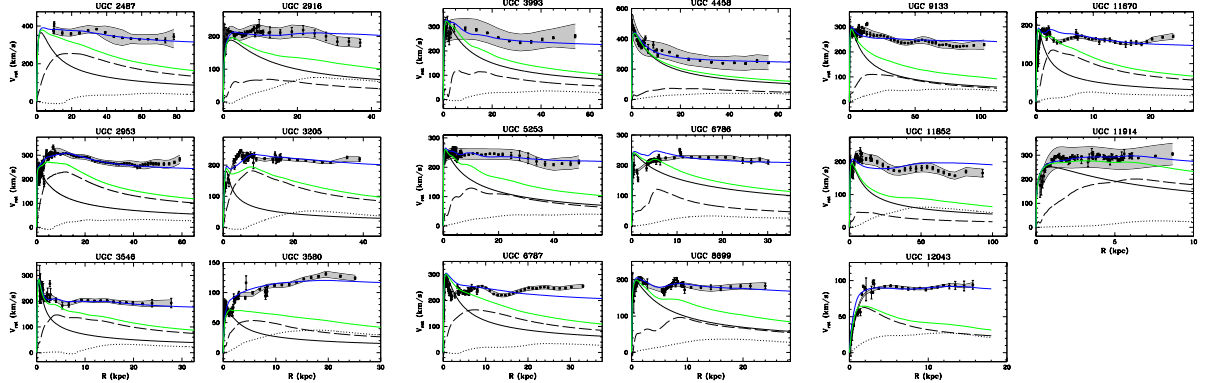


FIGURE 4. Additional MOND rotation curves from [74]. The grey shaded bands give the allowed range due to inclination uncertainties. Thin black solid, dashed and dotted lines give the contributions from stellar bulge, disc and gas respectively. The thin green (grey) line gives the Newtonian sum of the individual components and the bold blue (grey) lines gives the total MOND rotation curve.

reiterates the fact that a still undetected mass component exists in cluster cores, but does not pinpoint it as the putative particle DM.

The solar system

We know of two concrete solar system phenomena that MOND could impact. The first is the so called Pioneer anomaly: a yet unexplained constant acceleration of the two Pioneer spacecraft towards the sun, of a magnitude $\approx \bar{a}_0 \equiv 2\pi a_0$. While this anomaly may yet turn out to result from a mundane, underestimated systematics, and not from new physics, MOND does have the potential to explain such an anomaly. Because a similar anomaly in the motions of the planets can be ruled out by a large margin, a new-physics explanation must hinge on the differences

between the planetary orbits (bound, nearly circular) and those of the spacecraft (unbound, highly hyperbolic) (see [54] and references therein for details).

I have recently pointed out [57] that certain MOND theories predict an unavoidable MOND effect in the inner solar system (planetary zone) having to do with the galactic acceleration field in which the solar system is falling (the external-field effect of MOND). Near the sun—i.e. at distances much smaller than the MOND radius of the sun, $R_M \equiv (M_\odot G/a_0)^{1/2} \approx 8 \times 10^3 \text{ a.u.}$ —this appears as an anomalous quadrupole field with anomalous potential $\phi_{an} = (q/4)a_0 R_M^{-1}(2z^2 - x^2 - y^2)$. Here, the z -axis is in the direction of the galactic center, and $q \sim -0.1$ depends on the strength of the galactic acceleration field at the position of the sun, and on the form of the MOND extrapolating function of the theory. This anomaly causes very distinctive effects on planetary motions, such as an anomalous perihelion precession with characteristic dependence on the length of the semi-major axis and on its orientation relative to the galactic center's direction. This effect seems to be below present day detection capabilities, but not inaccessibly below.

MOND VS. COLD DARK MATTER

MOND and DM were initially advanced to explain similar phenomena: the mass discrepancies in galactic systems. This may create the impression that they are twin paradigms, that the deductions we make from both are of similar force, and that the only way to decide between them is to compare the performance of their predictions against the data in cases where the predictions differ. This is far from being the case.

I concentrate henceforth on cold dark matter (CDM), the eminently favored version of the DM paradigm today. Deductions made in CDM and in MOND, pertaining even to the same phenomena and measurements, such as galaxy rotation curves, the existence and nature of a TF relation, the distribution of the mass discrepancy in a given system, etc., are of a totally different nature. In MOND, these are strict and inevitable predictions of all aspects of the mass discrepancy in any given object, based only on the distribution of normal (baryonic) matter as it is now observed. Such predictions can be made for any given, individual system. In the CDM paradigm, all such deductions for a given object would depend strongly on its detailed history: initial collapse, merger history, cannibalism of satellites, gas accretion, star formation, baryon ejection by supernova explosions and ram pressure effects, dissipation, angular momentum exchange, cooling and heating, etc., etc.—details that we do not know, and cannot know for any individual object, with very few, important exceptions (see below). To make matters worse, the two major components, baryons and the putative, weakly interacting CDM, partake very differently in all the above processes. Despite many works in the literature that attempt to account for these via so called ‘semi analytic’ calculations, the bare fact is that CDM is incapable of truly predicting the mass discrepancies in galactic systems (again with that important exception to be discussed below). CDM is utterly incapable of predicting even the most basic number: the baryon fraction in a given galaxy.⁵ But such numbers are the preliminaries for arriving at a deduced TF relation for example, to say nothing of predicting full rotation curves. What we then see often as CDM analysis of rotation curves are merely best fits, which assume, in addition to the baryons, a certain DM halo mass distribution with the total mass and radial scale of the distribution left as free parameters. The MOND rotation curves have none of this. At worst they involve one free parameter for converting stellar light into baryonic mass (which CDM fits also have in addition to the other parameters). But even this is not needed in many instances, as explained above.

The other important point to note is that even if CDM is one day found capable of deducing from first principles some statistical, population properties pertaining to the relations between baryons and DM (such as the TF relation), it is highly unlikely to ever make predictions for individual systems, as MOND does.

To recapitulate, the fact that galaxies follow closely the MOND predictions argue against CDM in two ways. In the first place because they show that there is a paradigm that can predict all those observed regularities that CDM cannot predict. But, in addition, the fact that such strict regularities exist at all—even irrespective of the existence of MOND—argues against CDM, because the relations between the baryonic and DM components is expected to be highly haphazard in the CDM paradigm.

I mentioned above that there is one important exception to CDM's incapability of predicting baryon-to-DM relations. This concern so called tidal dwarfs: the small ‘phoenix’ galaxies that are born from the gaseous tidal tails

⁵ We know that this fraction in a galaxy today is typically much smaller than the cosmic fraction with which proto-galaxies must have started their life. So most of the baryons must have been lost on the way, somehow (in the CDM paradigm), exactly how, and what small fraction of it is left, are the grist for ‘semi analytic’ models.

spewed in the aftermath of high speed collisions between ready made galaxies. Such tidal dwarfs are an exception to the above rule because their history is brief and involves only a few, rather well understood processes. Simulations show that hardly any of the CDM purportedly surrounding the two colliding, parent galaxies makes its way into the tidal tails and the tidal dwarfs that form in them. CDM then predicts no mass discrepancies to speak of in tidal dwarfs. In MOND, the criterion for the appearance of a mass discrepancy is only the accelerations involved, not, in any way, the process of its formation.

So, what is known about mass discrepancies in tidal dwarfs? There are now two possible relevant instances. In the first, Bournaud et al. [18] have identified and analyzed three tidal dwarfs in the tidal debris around NGC 5291. They find, in contrast with the expectations from the CDM paradigm, that the three dwarfs do exhibit mass discrepancies of about a factor three. Subsequent analysis [52, 31] showed that these agree well with the predictions of MOND for these systems. In another possible instance, it has been argued recently [42] that most of the dwarf spheroidal satellites of the Milky Way are remnant tidal dwarfs in a single historical tidal tail. This deduction is based on the fact that both the position of these dwarfs and their motions indicate that they live and move in a disc around the Milky Way. This, and other aspects of the population, are difficult to understand unless, it is argued, they formed as tidal dwarfs. If so, the CDM paradigm implies they should not exhibit the large mass discrepancies they are known to show. In MOND, as said above, we do expect them to show large discrepancies because they are very low acceleration systems.

We saw that even the most common tests, such as RC analysis, hold a strong potential for deciding between MOND and CDM, for those who are willing to grasp better the differences in the nature of these deductions in MOND and CDM. However, in addition there are quite a few phenomena on which MOND and CDM make disparate predictions. I already discussed above the case in point concerning tidal dwarfs. A few other examples are: 1. MOND, but not CDM, predicts that in disc galaxies there will be a strong disc component of ‘phantom’ DM in addition to the extended spheroidal one. The predicted disc component appears only where the baryonic disc is, and only where the orbital accelerations in the disc are small in the MOND sense [50]. 2. Insistence on interpreting MOND results with DM will lead to negative DM densities in some locations [45]. 3. We saw already that some MOND theories predict an anomalous quadrupole field in the inner solar system, which would be quite unexpected from CDM.

To summarize this section: The success of MOND predictions implies that baryons alone strictly determine the acceleration fields of galactic objects. This conflicts with the expectations in the CDM paradigm (where these fields are, by and large, dominated by the contribution of the DM) because of the haphazard formation and evolution of galactic objects, and because baryons and DM are given to very different behaviors during the evolution. This is evinced by the very different characteristics of the baryonic and the putative DM components in galaxies today (e.g., the very small baryon-to-DM fraction in galaxies compared with the cosmic value, the highly concentrated, and oftentimes disc-like, baryonic component compared with the much more extended, spheroidal DM component). It is thus highly unlikely that DM will someday reproduce MOND.

THE COSMOLOGICAL CONNECTION

So far I discussed MOND only from the phenomenological point of view: It has been advanced to obviate the need for DM, and it has, indeed, managed to predict, explain, and organize large amounts of observations in individual systems, and of population laws and regularities, without invoking DM. However, there is an additional aspect revealed by MOND that may hint at more profound implications: The value of the MOND constant a_0 , as determined from the various laws listed above, is $a_0 \approx 1.2 \times 10^{-8} \text{cm s}^{-2}$. This value is tantalizingly close to acceleration parameters that characterize cosmology [43, 49]. Define, more conveniently, $\bar{a}_0 \equiv 2\pi a_0$, then one finds

$$\bar{a}_0 \approx a_H = cH_0, \quad \bar{a}_0 \approx a_\Lambda = c(\Lambda/3)^{1/2}. \quad (13)$$

The first acceleration parameter, a_H , is associated with the present day expansion rate H_0 , while the second is defined by the present density of ‘dark energy’ as determined by Λ . The mysterious fact that today the two cosmic acceleration parameters are nearly the same defines the ‘cosmic coincidence’ puzzle (another coincidence puzzle being the proximity of the baryon and DM densities, when the two components are thought to have formed by totally different and unrelated processes). The fact that \bar{a}_0 —an acceleration that emerges very clearly and forcibly from the phenomenology of galactic systems—also nearly equals these two cosmic accelerations should be viewed, in my opinion, as an added cosmic coincidence.

Much has been said about the significance of this last ‘coincidence’ and its possible origins and implications (e.g., in [43, 69, 49, 11, 12, 55, 59]). Here I only touch briefly on a recent observation that might connect MOND with

cosmology even more firmly [55]. If the ‘dark energy’ is a cosmological constant (CC), then our universe is nearly of a de Sitter (dS) geometry in the vicinity of the present time. In the future, as the CC becomes increasingly dominant over matter, the geometry will tend towards exact dS geometry. Now, the second coincidence in eq.(13) is tantamount to the MOND acceleration being related to the radius, ℓ , of this asymptotic dS cosmos by $\bar{a}_0 \approx c^2/\ell$. The possible tighter ties of MOND with cosmology may then be reflected in the symmetries that characterize these two structures. The symmetry (isometry) group of a 4-D dS space-time is the group of rotations in Minkowski 5-D space time; i.e. $SO(4, 1)$. If MOND, and in particular the deep MOND limit, is in some way a reflection of the near dS nature of our cosmos, the symmetries characterizing MOND might reflect the symmetries of the dS space-time, or perhaps both symmetries are a result of the symmetries of the theory that underlies both. Our actual universe is not quite of dS geometry; so we do not expect MOND at large to share symmetries with that geometry. Indeed MOND with its full coverage is not known to have symmetries beyond the standard rotation, and space- and time-translation invariance. But the deep MOND limit itself does have added symmetries in certain formulations. For example, the deep MOND limit of the nonlinear Poisson formulation eq.(3) has been found to have the full conformal symmetry in three space dimensions. Together with the space rotations and translations these conformal symmetries form a larger group which is equivalent to the $SO(4,1)$ geometrical symmetry group of exact dS space-time. This has been proposed [55] as a possible indication of a connection similar to the celebrated duality of conformal field theories on the boundary of an Anti-dS (or dS) space-time and gravity in that space-time. Here as well, the space on which the extended deep-MOND theories act is the Euclidean three-dimensional space (or rather its compactification to a Euclidean 3-D sphere), and this is indeed the past (or future) boundary of a dS space-time.

OPEN QUESTIONS

Beside the yet unexplained remaining factor-of-two discrepancy in galaxy clusters, the major challenge to MOND is to account fully for the need for cosmological DM. In other words we need a MOND inspired mechanism that will account for the effects in galaxy formation, CMB, etc., of what in standard dynamics is attributed to cosmological DM with a contribution of about 0.2 to Ω . One effect of cosmological DM is to enhance structure formation because, being neutral, it would have started to collapse before recombination. MOND clearly has the ability to enhance gravitational collapse over standard dynamics without DM. Several studies of structure formation in MOND inspired schemes, some using somewhat crude NR models, some using relativistic theories such as TeVeS, have been conducted, and have shown such enhancing effects: there are indeed aspects of MOND that could replace cosmological DM effects [70, 65, 82, 37, 81, 27, 92, 38]. It has to be realized, however, that the theory of structure formation, interaction of matter with the CMB, etc. in MOND is still much less developed and focussed then in standard dynamics. In my opinion a satisfactory understanding of all these will come only after we better understand the nature of the MOND-cosmology connection discussed above, and when we then have a full fledged relativistic version of MOND, which, arguably, we are still lacking.

In any event, one should certainly not be dismayed by the tasks still left to be accomplished. The very simple MOND idea proposed a quarter of a century ago, has already achieved much more then could be expected of it at the time of its inception. It can in fact be argued that the richness and variety, and sheer quantity of successful predictions in the context of galaxy dynamics is many fold what is still required in cosmology. Understanding a single galaxy is not as important as understanding cosmology. But, epistemologically, the detailed prediction of a field of a single galaxy—accounting exactly for what otherwise would result from a detailed amount and distribution of DM—is, in my opinion, a feat on a par with predicting the few statistical properties attributed conventionally to cosmological DM. After all, Newtonian dynamics was based on the observations of only one solar system.

All in all, with the encouragement from past successes and the attraction of remaining challenges, times are interesting for those who choose to work on MOND.

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